Biaxial Buckling Coefficients of Thin Rectangular Isotropic Plates, Having One Edge Simply Supported And The Other Edges Clamped.

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Abstract - This work was concerned with the application of the Tailor McLaurin's series in the determination of the buckling coefficients of thin rectangular SCCC plates loaded biaxially. The study was limited to thin rectangular isotropic plates, having aspect ratios between 1 and 2. The particular equation for the determination of the critical buckling load for the SCCC plate, was obtained by substituting the particular shape function (obtained via the Tailor-McLaurin's series) into the governing equation for the buckling of biaxially loaded plates. The result was obtained as the force in the x-direction in terms of that in the y-direction using a relationship constant-k which varied from 0.1 to 1. The numerical values of the aspect ratios and k, were substituted into the critical buckling load equation, to obtain the critical buckling load coefficients for each aspect ratio and k-value. The results showed that as the aspect ratios increased from 1 to 2 and as the k-values increased from 0.1 to 1, the buckling coefficients reduced respectively. Given that no results were found in the literature to compare with those of the present study, It was hence concluded that the results of this work are original.

Index Terms: Biaxial stresses, Buckling Coefficients, Thin Plate, In-plane forces, Tailor-Mclaurin's Series, Boundary conditions, aspect ratio.

1. INTRODUCTION:

Thin plates are initially flat structural members bounded by two parallel planes, called faces. The distance between the plane faces is called the thickness (h) of the plate. Usually, the plate thickness is small compared with other characteristic dimensions of the faces (length, width, diameter, etc.

Geometrically, plates are bounded either by straight or curved lines. Ventsel and Krauthammar [1], classified plates as thick (with $\frac{a}{h} \leq 10$.), thin (with $8 \leq \frac{a}{h} \leq 100$.) and Membranes (with $\frac{a}{h} \geq 80$). According to Szilard [2], thin plates can further be subdivided into two, namely: Stiff Plates **(**thin plates with flexural rigidity, which carries loads two dimensionally, mostly by internal bending and torsional moments and by transverse shear and having $\frac{h}{a} = \frac{1}{50} - \frac{1}{10}$ and Flexible plates (plates whose ratio of deflection to thickness, is beyond 0.3, and their lateral deflections, are accompanied by a stretching of the middle surface). The importance of this structural component and its wide application in different fields of Engineering such as, Civil, Mechanical, Structural, Aeronautic and Marine Engineering, has provoked many researches in the area of plate analysis. Plates support either lateral or in-plane loads. They support lateral loads by bending and in-plane loads by buckling. Buckling is a phenomenon where by a material under the action of compressive loads, passes from the state of stable equilibrium to a state of unstable equilibrium. The value of the compressive load at which this material passes from the stable to the unstable equilibrium, is called the critical load. This critical load, is the load that should not be reached in design. Hence, the determination of this load is very essential in engineering practice. This is to ensure designs which are within their safe limits. Several works

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have been done on the buckling of plates by other researchers. Wang and Long [3], used the finite element methods, to investigate the effect of hole shape, hole size and hole position on the elastic buckling of square perforated plates. Yang et.al [4], developed an analytical method to investigate the nonlinear buckling and expansion behaviors of local delamination near the surface of functionally graded laminated piezoelectric composite shells subjected to the thermal, electrical and mechanical loads, where the mid-plane nonlinear geometrical relation of delamination is considered. Ibearugbulem et.al [5], investigated the comprehensive buckling analysis of axially compressed rectangular flat thin plate with simply supported edges using a theoretical formulation based on the Taylor-McLaurin's series (truncated at the fifth term) on the Ritz method. Ibearugbulem et.al [6], investigated the use of Taylor-McLaurin series in the inelastic buckling analysis of a thin, flat, rectangular, isotropic plate bounded by four clamped edges, subjected to uniform uniaxial in-plane compression. They obtained the inelastic buckling behavior of the plate by adopting the deformation plasticity theory using Stowell's approach. Viswanathana et al. [7], used the quantic spline approximation technique to carry out the buckling of rectangular plates of variable thickness resting on an elastic foundation and subjected to an in-plane loads at two opposite edges. A polynomial shape function derived by Ibearugbulem [8], was used by Ibearugbulem et al [10] in the Ritz method to carry out the buckling analysis of plates with boundary conditions such as the SSSS, CCCC, CSSS, CCSS, CSCS, and the CCCS. From the foregoing, it is seen that works have not been done on the biaxial buckling of thin rectangular plates. Hence, this work aims at providing solutions for the biaxial buckling of thin rectangular isotropic SCCC plates, using a polynomial shape function given by Ibearugbulem [8], which is based on the tailor McLaurin's series.

2. EQUATION FOR THE DETERMINATION OF THE BUCKLING COEFFICIENTS OF THE PLATE.

Consider a thin rectangular plate bounded by x = 0, x = a, y = 0 and y = b, as shown in Figure 1. The plate is subjected to an in-plane compressive force Nx (acting on the normal to the edge x = 0 and x = a) and Ny (acting on the normal to the edge , y = 0 and y= b). Let the equation describing the buckling of the plates as stated in Onwuka et al [9] be Equation 1.

$$-N_x\left(\frac{\partial^2 w}{\partial x^2}\right) - N_y\left(\frac{\partial^2 w}{\partial y^2}\right) = D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \quad (1)$$

Where "w" is the deflection of the plate and D is the plate's flexural rigidity.

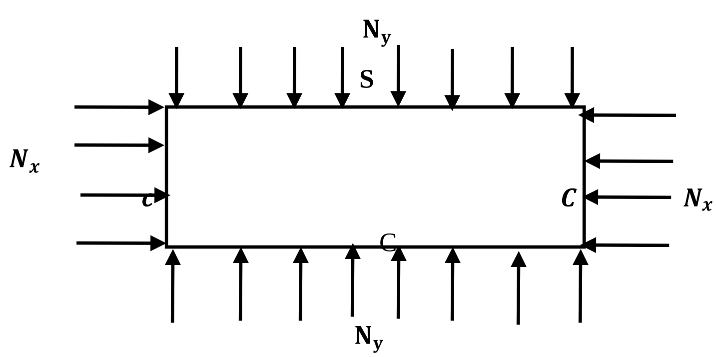

Fig 1: SCCC flat isotropic plate under biaxial in − plane loads

Upon the application of these compressive loads on the plates, it tends to deform from its initial state. Let the deflection of the plate due to the in-plane loads be "w". Hence, multiplying Equation 1 by the plate's deflection (w), yields the work equation of the plate as Equation 2.

$$-N_x\left(\frac{\partial^2 w}{\partial x^2}\right)w - N_y\left(\frac{\partial^2 w}{\partial y^2}\right)w = D\left[\frac{\partial^2 w}{\partial x^4}.w + 2w.\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right] \quad (2)$$

Let the deflection (w) of the plate, be as defined in Equation 3.

$$w = AH \quad (3)$$

Where "A" is the amplitude of deflection of the plate and H, the plate's particular shape function, given by Ibearugbulem [8] as Equation (3a)

$$H = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (3a)$$

Equation (3a), can be further broken down to its x and y- components -w_x *and* w_y as Equations (3b) and (3c) respectively.

$$w_x = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) \quad (3b)$$

$$w_y = (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (3c)$$

If we integrate Equation 2 indefinitely along the axes (x and y) of the plate, we will have the total work equation as Equation 4.

$$-N_x\iint\left(\frac{\partial^2 w}{\partial x^2}\right).w\partial x\partial y - N_y\iint\left(\frac{\partial^2 w}{\partial y^2}\right).w\partial x\partial y$$
$$= D\iint\left[\frac{\partial^4 w}{\partial x^4}.w + 2w\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right]\partial x\partial y \quad (4)$$

If R and Q are non-dimensional parameters of the plate on the x and y axes, respectively defined by Equation (5), and $\propto$ (the aspect ratio) defined by Equation (6),

$$R = \frac{x}{a} \text{ and } Q = \frac{y}{b} \quad (5)$$

$$\propto = \frac{b}{a} \quad (6)$$

Where "a" and "b" are the plate dimensions in the primary and secondary axes respectively,

Then substituting Equations (3), (5) and (6) into Equation (4), yields Equation (7)

$$-\frac{N_x}{a^2}\iint\left(\frac{\partial H}{\partial R}\right)^2\partial R\partial Q - \frac{N_y}{a^2\propto^2}\iint\left(\frac{\partial H}{\partial Q}\right)^2\partial R\partial Q$$
$$= \frac{D}{a^4}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2\partial Q^2}\right) + \frac{H}{\propto^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R\partial Q \quad (7)$$

If we let Ny and Nx to be related by Equation (8),

$$N_y = KN_x \quad (8)$$

Then substituting Equation (8) into Equation (7), and multiplying the outcome by the square of the primary dimension of the plate (a^2), will yield Equation (9) hence;

$$-\frac{N_x}{a^2}\iint\left(\frac{\partial H}{\partial R}\right)^2\partial R\partial Q-\frac{KN_x}{a^2\propto^2}\iint\left(\frac{\partial H}{\partial Q}\right)^2\partial R\partial Q$$
$$=\frac{D}{a^4}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H+\frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2\partial Q^2}\right)+\frac{H}{\propto^4}\left(\frac{\partial^4 H}{dQ^4}\right)\right]\partial R\partial Q \quad (9)$$

Making the in-plane load (Nx) on the x-axis of the plate the subject of Equation (9), yields Equation (10), which is the general equation of buckling of a biaxially compressed thin rectangular isotropic plates.

$$\boldsymbol{N_x}=-\frac{{}^{\boldsymbol{D}}/_{\boldsymbol{a}^2}\iint\left[\left(\frac{\partial^4 H}{\partial R^4}\right)H+\frac{2H}{\propto^2}\left(\frac{\partial^4 H}{\partial R^2\partial Q^2}\right)+\frac{H}{\propto^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R\partial Q}{\iint\left[\left(\frac{\partial H}{\partial R}\right)^2+\frac{K}{\propto^2}\left(\frac{\partial H}{\partial Q}\right)^2\right]\partial R\partial Q} \quad (10)$$

3. DETERMINATION OF THE SHAPE FUNCTION OF THE SCCC RECTANGULAR PLATE.

The boundary conditions of the SCCC plates are as given below,

At

$R=0, \quad w=w'=0$ (11)

$R=1, \quad w=w'=0$ (12)

$Q=0, \quad w=w'=0$ (13)

$Q=1, \quad w=w''=0$ (14)

Where w' and w" are the first and second derivatives of the plate's deflection equation. Given that the plate is treated as strands of beams in the x and y axes, let us consider the x axis. The first derivative of Equation 3b with respect to R, gives Equation (15).

$$w'=a_1+2a_2R+3a_3R^2+4a_4R^3 \quad (15)$$

Substituting the boundary conditions into Equation (3b) and Equation (15) respectively, yields, Equations (16) and (17) at R=0 and Equations (18) and (19) at R=1.

$w(0)=0\Longrightarrow a_0=0$ (16)

$w'(0)=0\Longrightarrow a_1=0$ (17)

$w(1)=0\Longrightarrow a_2+a_3+a_4=0$ (18)

$w'(1)=0\Longrightarrow 2a_2+3a_3+4a_4=0$ (19)

Solving and simplifying Equation (18) and Equation (19) simultaneously, yields Equations (20) and (21)

$a_3=-2a_4$ (20)

$a_2=a_4$ (21)

Substituting Equations (16), (17), (20), and Equations (21) into Equation (3b), yields the X component of the deflection equation of the SCCC plate as Equation (22),

$$w_x = a_4(R^2 - 2R^3 + R^4) \tag{22}$$

Similarly, carrying out (in the y direction) the same procedure outlined in Equations 15-21, yields the y component of the deflection equation as Equation (23)

$$W_y = b_4(0.5Q - 1.5Q^3 + Q^4) \tag{23}$$

Substituting Equations (22) and (23) into Equation (3), gives the deflection equation of the plates as Equation (24)

$$w = a_4 b_4 (R^2 - 2R^3 + R^4)(0.5Q - 1.5Q^3 + Q^4) \tag{24}$$

Where $a_4 b_4 = A$ (25)

and "H", the shape function, is given by Equation (26)

$$H = (R^2 - 2R^3 + R^4)(0.5Q - 1.5Q^3 + Q^4) \tag{26}$$

4. DETERMINATION OF THE BUCKLING COEFFICIENTS OF THE SCCC PLATES.

Differentiating Equation (26) with respect to R and Q and taking the square the outcome, gives the shape function derivatives as presented in Equations (27) and (28) while multiplying the derivatives with the shape function (H), gives the results presented in Equations (29) to (31) hence

$$\left(\frac{\partial H}{\partial R}\right)^2 = (4R^2 - 24R^3 + 52R^4 - 48R^5 + 16R^5)(0.25Q^2 - 1.5Q^4 + Q^5 + 2.25Q^6 - 3Q^7 + Q^8) \tag{27}$$

$$\left(\frac{\partial H}{\partial Q}\right)^2 = (R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)\ (0.25 - 4.5Q^2 + 4Q^3 + 20.25Q^4 - 36Q^5 + 16Q^6) \tag{28}$$

$$\left(\frac{\partial^4 H}{\partial R^4}\right)H = [24(R^2 - 2R^3 + R^4)(0.25Q^2 - 1.5Q^4 + Q^5 + 2.25Q^6 - 3Q^7 + Q^8)] \tag{29}$$

$$\left(\frac{\partial^4 H}{\partial Q^4}\right)H = 24(R^4 - 4R^5 + 6R^6 - 4R^7 + R^8)(0.5Q - 1.5Q^3 + Q^4) \tag{30}$$

$$\left(\frac{\partial^4 H}{\partial R^2 \partial Q^2}\right)H = (2R^2 - 16R^3 + 38R^4 - 36R^5 + 12R^6)\ (-4.5Q^2 + 6Q^3 + 13.5Q^4 - 27Q^5 + 12Q^6) \tag{31}$$

Integrating Equations (27) to (31), between 0-1, yields the results given in equations (32) - (36)

$$\int_0^1 \int_0^1 \left(\frac{\partial H}{\partial R}\right)^2 \partial R \partial Q = 1.436130004 * 10^{-4} \tag{32}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial H}{\partial Q}\right)^2 \partial R \partial Q = 1.360544217 * 10^{-4} \tag{33}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^4 H}{\partial R^4}\right) H \partial R \partial Q = 6.031745972 * 10^{-3} \tag{34}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^4 H}{\partial R^2 \partial Q^4}\right) H \partial R \partial Q = 1.632653047 * 10^{-3} \tag{35}$$

$$\int_0^1 \int_0^1 \left(\frac{\partial^4 H}{\partial Q^4}\right) H \partial R \partial Q = 2.857142857 * 10^{-3} \tag{36}$$

Substituting the values obtained in Equations (32) to (36) into Equation (10), yields Equation (37)

$$N_x = -\frac{\frac{D}{a^2}\left[6.03174597 * 10^{-3} + \frac{2}{\propto^2}(1.63265304 * 10^{-3}) + \frac{1}{\propto^4}(2.857142857 * 10^{-3})\right]}{\left[1.436130004 * 10^{-4} + \frac{k}{\propto^2}(1.360544217 * 10^{-4})\right]} \tag{37}$$

Equation (37), is the particular equation for the determination of the critical buckling loads of a biaxially loaded SCCC plate. The buckling load coefficients of the plate can be obtained from Equation (38)

$$F = -\frac{\left[6.03174597 * 10^{-3} + \frac{2}{\propto^2}(1.63265304 * 10^{-3}) + \frac{1}{\propto^4}(2.857142857 * 10^{-3})\right]}{\left[1.436130004 * 10^{-4} + \frac{k}{\propto^2}(1.360544217 * 10^{-4})\right]} \tag{38}$$

If we substitute the range of numerical values (1 to 2) for $\propto$ and (0.1 to 1) for k, into equation (38), we will have the results for the critical buckling load coefficients of a biaxially loaded SCCC plates as shown in Table 1

Table 1: Critical Buckling Load coefficients for an SCCC plate loaded biaxially.

Aspect Ratio	CRITICAL BUCKLING LOAD COEFFICIENTS (Nxi)										
	N_x	N_{x1}	N_{x2}	N_{x3}	N_{x4}	N_{x5}	N_{x6}	N_{x7}	N_{x8}	N_{x9}	N_{x10}
1	84.63158	77.30769	71.15044	65.90164	61.37405	57.42857	53.95973	50.88608	48.14371	45.68182	43.45946
1.1	74.37915	68.97849	64.30901	60.23165	56.6405	53.45348	50.606	48.04655	45.73353	43.63299	41.71692
1.2	67.38377	63.22428	59.54845	56.27656	53.34549	50.70462	48.31289	46.13664	44.14799	42.32369	40.64418
1.3	62.41945	59.10612	56.12682	53.43345	50.98674	48.75428	46.70913	44.82864	43.09371	41.48807	39.99778
1.4	58.77919	56.06909	53.59787	51.3353	49.25601	47.33861	45.56489	43.91928	42.3884	40.96065	39.62594
1.5	56.03509	53.77104	51.68285	49.75078	47.95796	46.28985	44.73389	43.27913	41.91601	40.63613	39.4321
1.6	53.91727	51.99318	50.20169	48.52954	46.96519	45.49855	44.12073	42.82391	41.60115	40.44627	39.35378
1.7	52.24942	50.591	49.03462	47.57115	46.19249	44.8915	43.66178	42.49764	41.39397	40.34617	39.3501
1.8	50.91271	49.46633	48.09986	46.80685	45.58154	44.41874	43.3138	42.2625	41.26101	40.3059	39.394
1.9	49.82489	48.55078	47.3402	46.18853	45.09156	44.04548	43.04685	42.09249	41.17953	40.30533	39.46748
2	48.92763	47.79563	46.71482	45.68182	44.69351	43.74706	42.83986	41.96953	41.13385	40.3308	39.55851

Where $N_{xi}(i = 1,2,3 \ldots 10)$, are the critical buckling load coefficients at $k = 0.i$.

Table 1 shows that, as the aspect ratios increase from 1 to 2 (for all k- values), and as the k-values increases (for a given aspect ratio), the buckling coefficients reduce consistently. This is due to the fact that, for a given k-value, when the aspect ratios are increased gradually from 1 to 2, the plate gradually becomes a one-way plate, losing its square shape and taking up an oblong shape, thus behaving as a slender column which is weak in buckling. Thus the buckling load reduces. As the k-value increases for a particular aspect ratio, the loads applied in the y-direction of the plate increases, thus, increasing the total load applied on the plate. In such a case, the plate's ability to resist the applied loads is reduced and hence, it buckles faster than it should have buckled without a corresponding force on the y-direction. This accounts for the reduction in the plates buckling load as the k-values (forces in the y-axis of the plate) are increased.

Literature was consulted in order to match the results of this research with existing results, but no existing result was found on this plate boundary condition.

5. CONCLUSION:

From this work, the following conclusions have been arrived at;

i. The equation for the determination of the biaxial critical buckling load for an SCCC plate has been derived.
ii. The critical buckling load coefficients for a biaxially loaded SCCC plate (for different aspect ratios and k-values), have been determined.
iii. This work is original, as there are no works in the literature to compare its results with.

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